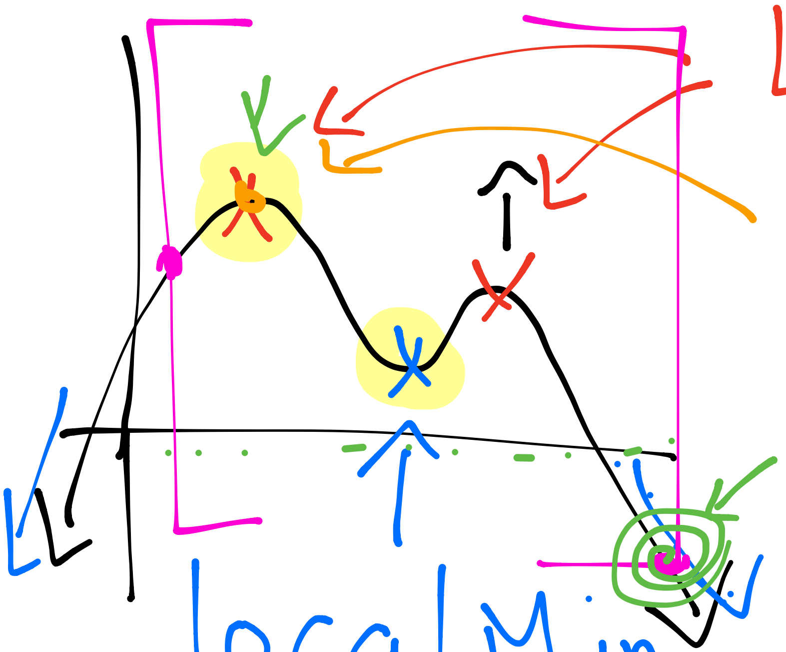
Section 4.2 Optimization

Suppose p is a point in the domain of f:

- f has a **global minimum** at p if $f(p)$ is less than or equal to all values of f.
- f has a **global maximum** at p if $f(p)$ is greater than or equal to all values of f.



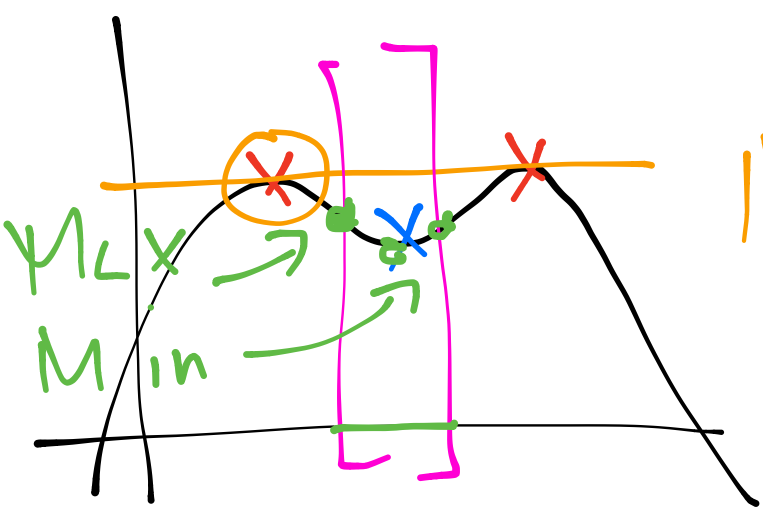
Local Max

Global Max

Local Min

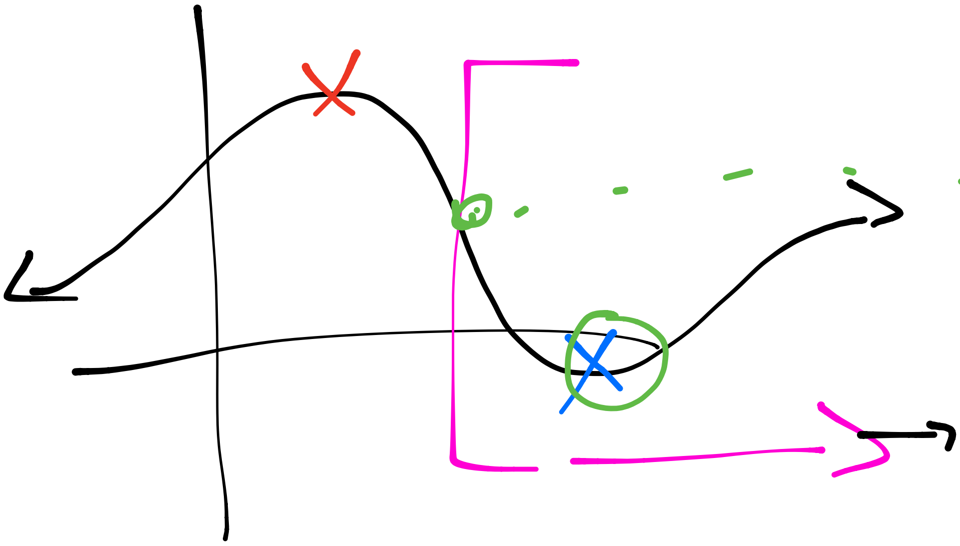
Not a Global Minimum.

No Global Min.



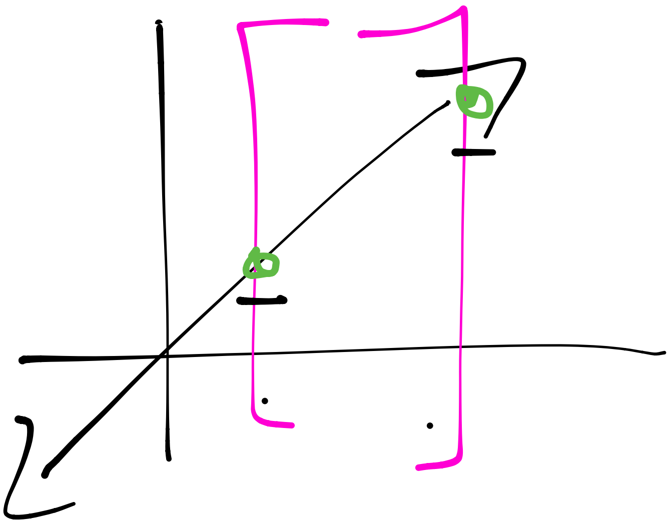
Global Maxima?

Two!



One of Each

☆ → open



No Global or Local Extrema

If I only consider a closed interval, Then there is ALWAYS a Global Max + Min.

Theorem 4.2: The Extreme Value Theorem

If f is continuous on the closed interval $a \leq x \leq b$, then f has a global maximum and a global minimum on that interval.

To find the Global Extrema we check:

① The y-value of all crit. pts. (in the interval) <-

② The y-value of the end pts.

End video 1

Ex: Let $f(x) = \dfrac{x^2+9}{x+4}$ ←

Find the global Extrema on $[-1, 6]$ ←

→ $f(x)$ is undefined at $x = -4$

Let Crit Pts. where is $f'(x) = 0$ or undefined?

$$f'(x) = \frac{[2x](x+4) \overset{*}{-} (x^2+9)[1]}{(x+4)^2}$$

$$= \frac{2x^2 + 8x - x^2 - 9}{(x+4)^2}$$

$$= \frac{x^2 + 8x - 9}{(x+4)^2} \leftarrow$$

$\boxed{= 0?}$

DNE?

$x = -4$ ✓

$x^2+8x-9=0$

9

→ 1, 9

→ 3, 3

FACTOR!

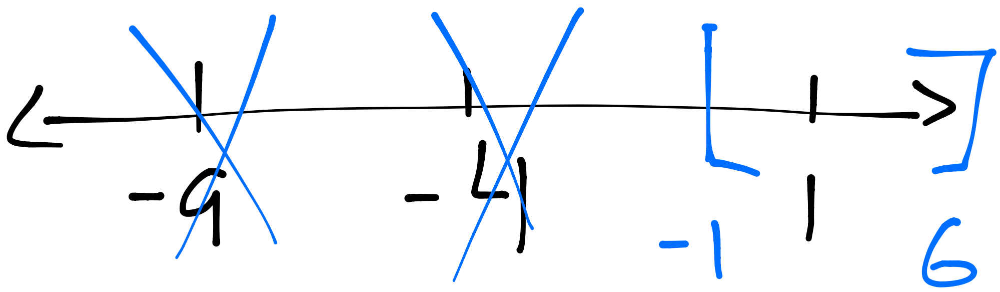
$(x+9)(x-1)=0$

$\Rightarrow x+9=0$ $\qquad$ $x-1=0$

$\boxed{x=-9}$ $\qquad$ $\boxed{x=1}$

Not in Domain → $\boxed{x=-4}$ So, Not a crit. pt.

Crit. pts:



To check: $x = -1, 1, 6$

Plug these into $f(x)$.

$$f(x) = \frac{x^2+9}{x-4} \rightarrow \frac{x^2+9}{x+4}$$

Wrong in Video
Corrected in Red

$$f(-1) = \frac{10}{-5} = \cancel{-2} = \frac{10}{3}$$

$$f(1) = \frac{10}{-3} = \cancel{-3\tfrac{1}{3}} = \frac{10}{5} = 2$$

$$f(6) = \frac{45}{2} = \cancel{22\tfrac{1}{2}} = \frac{45}{10} = 4.5$$

The global Max is at $x=6$ and $y=22.5$

Global Min $x=1$, $y=-3\tfrac{1}{3}$.