Calc I - §4.1 Using the Derivative

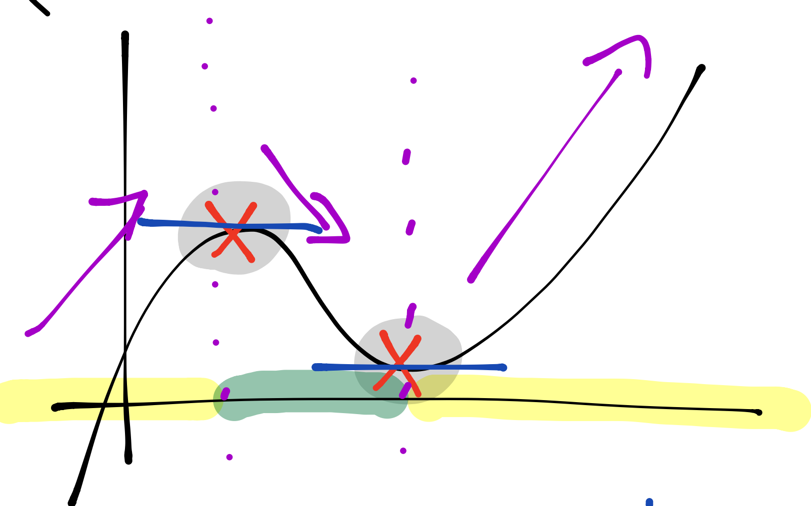
flat cusp

A **critical point** of a function *f* is a point *p* in the domain of *f* such that *f'(p) =0* or is undefined.

The function *f* has a **local maximum** at a point *p* if *f(p)* is greater than or equal to all the values of *f* near *p*.

And **local minimum** is the same but less than or equal.

Last, a point *p* where the function *f* changes concavity, is called an **inflection point**.



$f'(x) = 0$

$\wedge \to$ undef.

If we know All the Crit. pt. then Everywhere else is Inc. or Dec.

The First-Derivative Test for Local Maxima and Minima

Suppose p is a critical point of a continuous function f. Moving from left to right:

- If f' changes from negative to positive at p, then f has a local minimum at p.
- If f' changes from positive to negative at p, then f has a local maximum at p.

Ex: Find all the local Extrema of

$f(x) = x^3 - 6x^2 + 2$.

$f'(x) = 3x^2 - 12x$

$f'(x) = 0$ or undefined ✓

$\hookrightarrow 3x^2 - 12x = 0$

Factor!

$(3x) \cdot [x - 4] = 0$

$3x = 0$ $\qquad\qquad$ $x - 4 = 0$

$x = 0$ $\qquad\qquad$ $x = 4$

$\boxed{\text{Crit. pts: } x = 0, 4}$

These are the possible spots for Extrema.

Test pts around the critical pts

$f'(x) = 3x^2 - 12x = 3x[x-4]$

x:	-1	0	1	4	100
$f'(x)$:	$+$	0	$-$	0	$+$
$f(x)$:	$\nearrow$	Max	$\searrow$	Min	$\nearrow$

The ftn f has a local Max at $x=0$ and a local Min at $x=4$.

Inc: $(-\infty, 0)$, $(4, \infty)$

Dec: $(0, 4)$.

End of First Video

The Second-Derivative Test for Local Maxima and Minima

- If $f'(p) = 0$ and $f''(p) > 0$ then f has a local minimum at p.
- If $f'(p) = 0$ and $f''(p) < 0$ then f has a local maximum at p.
- If $f'(p) = 0$ and $f''(p) = 0$ then the test tells us nothing.

From last Video:

$f(x) = x^3 - 6x^2 + 2$

$f'(x) = 3x^2 - 12x$

crit pts.

L. Min at $\boxed{x = 4}$

L. Max at $\boxed{x = 0}$.

Plug into $f''(x)$.

$f''(x) = 6x - 12$

$f''(0) = -12$ $\Downarrow$ Local Max.

$f''(4) = 12$ $\Downarrow$ Local Min.

Inflection Pts $\Rightarrow$ Concave Up $\rightleftarrows$ Down.

Find the possible spots

$f''(x) = 0$ or Undefined ✓

$$f''(x) = 6x - 12$$

Defined Everywhere.

$$f''(x) = 0$$

$$6x - 12 = 0$$

$$x = 2$$

x:	0	2	4
$f''(x)$:	$-$	0	$+$
$f(x)$:	$\frown$	$\uparrow$	$\smile$

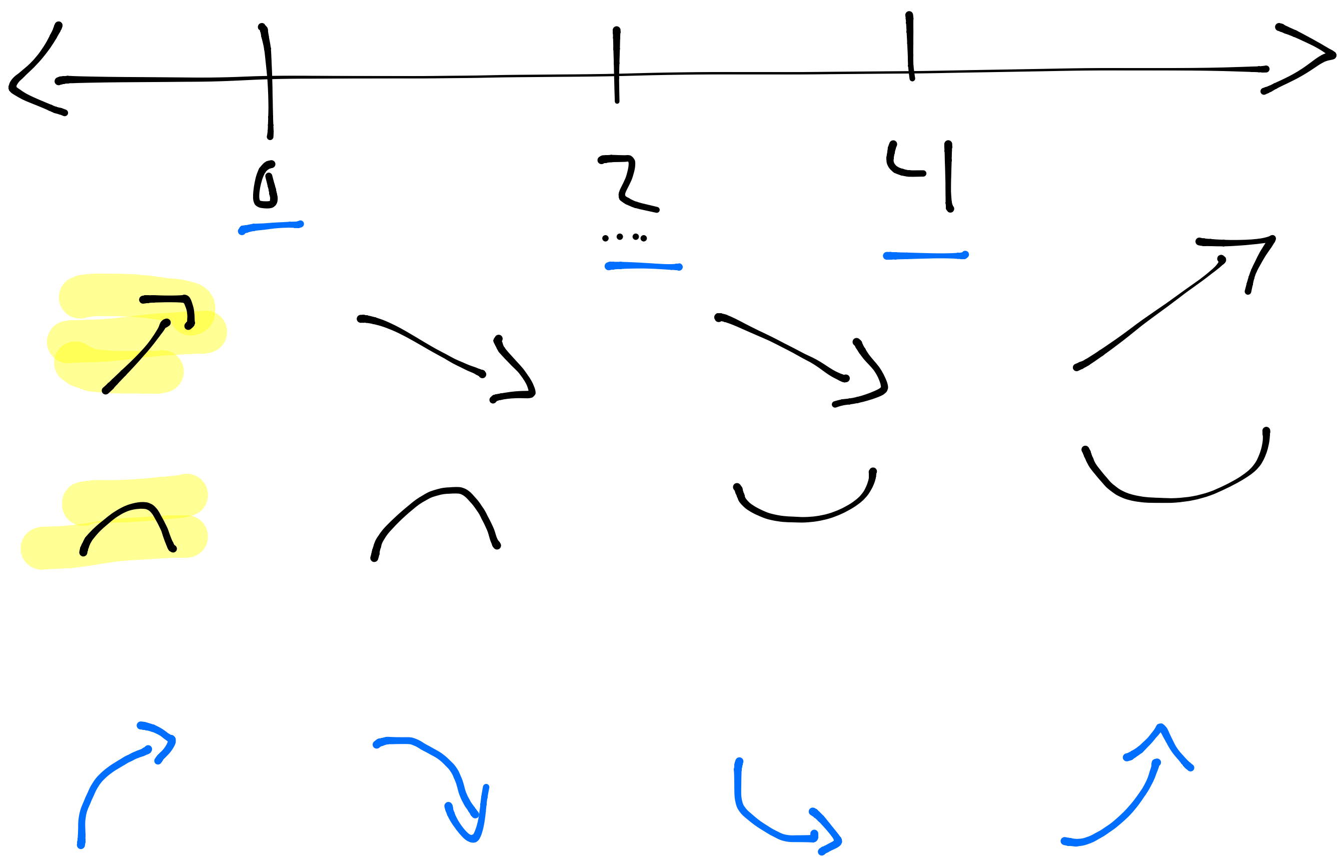
Infl. Pt.

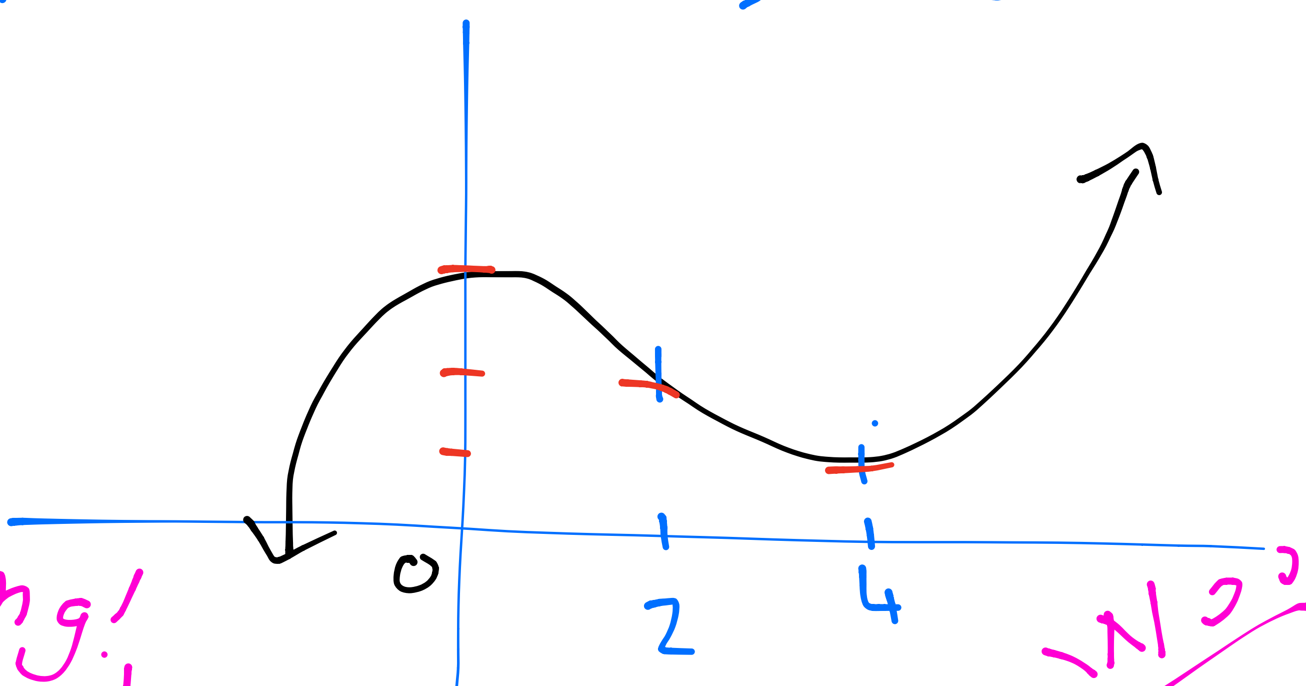
$f(x)$ has an Inflection Pt at $x = 2$.

$f(x)$ is concave up on $(2, \infty)$

Concave Down $(-\infty, 2)$.

Sketch the graph.





Zomg! Hi, guys!

Woo!