Name: ______________________________ MA 253 - Section C

Section 4.4 Example April 24, 2020

This is to supplement the video, which has poor quality video, making the example impossible to read.

Example: $\mathbb{P}_3$ is the set of polynomials of degree at most 3. So stuff that can be written

$$a_0 + a_1 t + a_2 t^2 + a_3 t^3.$$

The two bases for $\mathbb{P}_3$ are

$$\mathcal{S} = \{1, t, t^2, t^3\}$$

$$\mathcal{T} = \{1, t+1, t^2+t+1, t^3+t^2+t+1\}$$

These are both bases because they are linearly independent and span the whole set.

The polynomial is $p = p(t) = 1 + t + t^2 + t^3$. We want to find $[p]_{\mathcal{S}}$, the coordinate of p relative to basis $\mathcal{S}$. We need to figure out how we add up the basis elements to get p. Well it is easy to see that we need one of each of the elements in $\mathcal{S}$.

$$[p]_{\mathcal{S}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

And now we do the same for $\mathcal{T}$. But this is easier, since p is literally one of the elements, just written in the reserve order (which doesn't matter). So

$$[p]_{\mathcal{T}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Now we do it again with a new polynomial. Let $q = t^3$. Then

$$[q]_{\mathcal{S}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

since t^3 is the last element of the basis $\mathcal{S}$. And for the other basis,

$$[q]_{\mathcal{T}} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

as $0(1) + 0(t+1) - 1(t^2+t+1) + 1(t^3+t^2+t+1) = t^3 = q$. That is the example worked out. **Try:** Let $r = 2 - t + 3t^3$. Find $[r]_{\mathcal{S}}$ and $[r]_{\mathcal{T}}$.