

Definition

If V is spanned by a finite set, then V is said to be **finite-dimensional**, and the **dimension** of V , written as $\dim V$, is the number of vectors in a basis for V . The dimension of the zero vector space $\{0\}$ is defined to be zero. If V is not spanned by a finite set, then V is said to be **infinite-dimensional**.

Thm 9

If a vector space V has a basis $\mathcal{B} = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$, then any set in V containing more than n vectors must be linearly dependent.

Thm 10

If a vector space V has a basis of n vectors, then every basis of V must consist of exactly n vectors.

If $\dim V = n$, then...

(Switch the highlighted text.)

Thm 11

Let H be a subspace of a finite-dimensional vector space V . Any linearly independent set in H can be expanded, if necessary, to a basis for H . Also, H is finite-dimensional and

$$\dim H \leq \dim V$$

Thm 12

The Basis Theorem

Let V be a p -dimensional vector space, $p \geq 1$. Any linearly independent set of exactly p elements in V is automatically a basis for V . Any set of exactly p elements that spans V is automatically a basis for V .

Give me p -elements
(in $\dim V = p$),

- If they are L.I.

\Rightarrow Basis

- If they span V

\Rightarrow Basis

